

**GUJARAT TECHNOLOGICAL UNIVERSITY****BE SEM-III Examination May 2012****Subject code: 130001****Subject Name: Mathematics - III****Date: 14/05/2012****Time: 02.30 pm – 05.30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

**Q.1 (a) Attempt all questions: 04**

(1) Solve the differential equation  $xy \frac{dy}{dx} = 1 + x + y + xy$

(2) Find the general solution of  $\frac{d^4 y}{dx^4} - 18 \frac{d^2 y}{dx^2} + 81y = 0$

(3) Find particular solution of  $y = \frac{1}{(D+1)^2} \cosh x$ , where  $D = \frac{d}{dx}$

(4) Find the value of  $\Gamma \frac{1}{4} \Gamma \frac{3}{4}$

**(b) Attempt the following equations: 10**

(1) Determine the singular points of differential equation  $2x(x-2)^2 y'' + 3xy' + (x-2)y = 0$  and classify them as regular or irregular.

(2) Find half range cosine series for  $f(x) = e^x$  in  $(0,1)$ .

(3) Find the Fourier sine transform of  $f(x) = e^{-2x} + e^{-3x}, x > 0$ .

(4) Solve:  $(x+y)^2 \left[ x \frac{dy}{dx} + y \right] = xy \left[ 1 + \frac{dy}{dx} \right]$  Evaluate :

(5)  $\int_0^1 x^4 \cos^{-1} x dx$

**Q.2 (a) Attempt the following questions:**

(1) Find the Laplace transform of  $f(t) = t^2 \sinh at$  **02**

(2) Find the Laplace transform of  $f(t) = \begin{cases} 0, 0 < t < \pi \\ \sin t, t > \pi \end{cases}$  **02**

(3) Find the inverse Laplace transform of  $\frac{5s+3}{(s^2+2s+5)(s-1)}$  **03**

**(b) Attempt the following questions :**

(1) Solve the differential equation:  $(x^2 y^2 + 2) y dx + (2 - x^2 y^2) x dy = 0$ . **03**

(2) Find the solution of differential equation  $y'' - 5y' + 6y = 0$  with initial condition  $y(1) = e^2$  and  $y'(1) = 3e^2$ . **02**

- (3) Find the Laplace transform of  $\frac{(1 - \cos t)}{t}$  02

OR

- (b) Attempt the following questions:

- (1) Using Laplace transform solve the differential equation 03

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t} \sin t \quad \text{where } x(0) = 0 \text{ and } x'(0) = 1.$$

- (2) Find the series solution of  $(1 + x^2)y'' + xy' - 9y = 0$ . 04

- Q.3** (a) Attempt the following questions

- (1) Solve:  $\frac{d^4y}{dt^4} - 2\frac{d^2y}{dt^2} + y = \cos t + e^{2t} + e^t$  03

- (2) Solve:  $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = \frac{e^{2x}}{x^5}$  03

- (3) The Bessel equation of order zero is  $x^2y'' + xy' + x^2y = 0$  then 04

(i) find the roots of the indicial equation

(ii) show that one solution for  $x > 0$  is  $y = c_0 J_0(x)$

$$\text{where, } J_0(x) = 1 + \sum \frac{(-1)^n x^{2n}}{2^{2n} (n!)^2}$$

- (b) Find fourier series for  $f(x) = \begin{cases} -\pi, & -\pi \leq x \leq 0 \\ x, & 0 \leq x \leq \pi \end{cases}$  04

$$\text{and show that } \frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$$

OR

- Q.3** (a) Attempt the following questions

- (1) Solve:  $\frac{d^2y}{dx^2} - \frac{d^2y}{dx^2} + \frac{3dy}{dx} + 5y = e^x \cos 3x$  03

- (2) Solve:  $(1+x)^2 \frac{d^2y}{dx^2} + (1+x) \frac{dy}{dx} + y = 4 \cos(\log(1+x))$  03

- (3) Find the series solution using by Fobenius method  $xy'' + y' - y = 0$  04

- (b) Find fourier series for  $f(x) = 2x - x^2$  in the interval  $(0,3)$ . 04

- Q.4** (a) Attempt the following questions :

- (1) Solve the differential equation  $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2e^x \cos\left(\frac{x}{2}\right)$  Solve the 03

- differential equation  $(X^2 D^2 - 3XD + 4)y = x^2$  given that  $y(1) = 1$  and  $y'(1) = 0$ . 03

- (3) Evaluate :  $\int_3^7 (x-3)^{1/4} (7-x)^{1/4} dx$  02

- (b) Attempt the following questions:

- (1) Prove that in usual notation 03

$$4J_n''(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$$

- (2) Find Laplace transform of (i)  $e^{-3t}u(t-2)$ , (ii)  $\int_0^t e^{-u} \cos u du$  03

OR

**Q.4 (a)** Attempt the following questions:

(1) Solve the differential equation  $\frac{d^3 y}{dx^3} + \frac{dy}{dx} = \operatorname{cosec} x$  by method of variation of parameters. **03**

(2) Solve :  $(D^2 - 4D + 4)y = \frac{e^{2x}}{1+x^2}$  where  $D = \frac{d}{dx}$  **03**

(3) Evaluate :  $\int_0^1 (x \log x)^3 dx$  **02**

**(b)** Attempt the following equation:

(1) Solve the differential equation  $\frac{d^2 y}{dt^2} + 4y = f(t)$ ,  $y(0) = 0$ ,  $y'(0) = 1$  by laplace transform **03**

where (i)  $f(t) = \begin{cases} 1, & 0 < t < 1 \\ 0, & t > 1 \end{cases}$  (ii)  $f(t) = H(t-2)$

(2) Find the fourier transform of  $e^{-\frac{x^2}{2}}$  is  $e^{-\frac{\lambda^2}{2}}$  **03**

**Q.5 (a)** Attempt the following equation: **05**

(1) Find half Range cosine series for  $\sin x$  in  $(0, \Pi)$  and show that

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\Pi}{4} \text{ And using parseval's Identity prove that}$$

$$\frac{1}{1^2 \cdot 3^2} + \frac{1}{3^2 \cdot 5^2} + \frac{1}{5^2 \cdot 7^2} + \dots = \frac{\Pi^2 - 8}{16}$$

(2) Solve  $\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$  by the method of separation of variables **04**

**(b)** A tightly stretched string with fixed end points  $x = 0$  and  $x = L$  is initially **05**

Given the displacement  $y = y_0 \sin^3\left(\frac{\Pi x}{L}\right)$  If it is released from rest from this position then find the displacement  $y$

use the equation  $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$

OR

**Q.5 (a)** Attempt the following equation: **05**

(1) If  $f(x) = \begin{cases} mx, & 0 \leq x \leq \frac{\Pi}{2} \\ m(\Pi - x), & \frac{\Pi}{2} \leq x \leq \Pi \end{cases}$  then show that **05**

$$f(x) = \frac{4m}{\Pi} \left\{ \frac{\sin x}{1^2} - \frac{\sin 3x}{3^2} + \frac{\sin 5x}{5^2} - \dots \right\}$$

(2) Determine the solution of one dimensional heat equation **04**

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \text{ where the boundary condition}$$

are  $u(0, t) = u(L, t) = 0, t > 0$  and the initial condition is

- $u(x,0) = x$ ,  $L$  being the length. ( $0 < x < L$ )
- (b) Solve the equation  $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$  for the condition of heat along a rod without radiation subject to the condition (i)  $\frac{\partial u}{\partial x} = 0$  for  $x = 0$  and  $x = L$
- (ii)  $u = Lx - x^2$  at  $t = 0$  and for all  $x$

05

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